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SYMMETRY VIOLATING KAON DECAYS

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ABSTRACT

The content of this talk comprises two parts. In the first, an analysis of the muon number violating decay modes of the K-mesons is given. Subsequently, some new developments in the field of CP-violation are reviewed and the question of time-reversal invariance and the status of CPT-invariance are briefly considered.

INTRODUCTION

The system of K-mesons has a remarkable record in the history of modern physics. In 1956, the famous "θ-τ puzzle" led to the discovery of parity-violation in the weak interactions. Eight years later, observation of the decay $K_L \rightarrow 2\pi$ destroyed the notion that CP-invariance is an exact symmetry of nature. More recently, the absence of any appreciable strangeness-changing neutral current interactions, indicated by the strong suppression of decays such as $K_L \rightarrow \mu^+\mu^-$, demanded in the framework of unified gauge theories the introduction of the charmed quark.[1] Subsequently, the order of magnitude of the charmed quark mass was successfully predicted from the observed K_L-K_S mass difference.[2]

Instrumental to this role were the relatively large mass of the K-mesons which allows a great variety of decay modes, including some nonleptonic ones, and the existence of two distinct, almost degenerate, neutral kaon states. Is it conceivable that studies of K-decays would lead to developments of similar importance in the future? The areas that appear to have the best chance are again those where an apparent symmetry principle would be probed. In this talk, I would like to consider some topics which belong to this domain. This is not to suggest that other aspects of K-decays are of minor importance. Precision measurements of the "classic" kaon (and hyperon) decays, for example, have become especially important at the present time, since new theoretical developments suggest some deviations from the Cabibbo model.[3] Also important are detailed studies of nonleptonic decays, to understand the pattern of violation of the $\Delta I = 1/2$ rule. Last, but not least, investigations of the "non-exotic" rare decays, such as $K_L \rightarrow \mu^+\mu^-$, $K^+ \rightarrow \pi^+\bar{\nu}\nu$,...are of great value since here one probes the effects of the higher order weak and electromagnetic interactions, calculable in principle in renormalizable gauge theories,[4] and also the possible presence, at some level, of strangeness-changing neutral currents.

MUON NUMBER VIOLATING KAON DECAYS

Recently there has been considerable interest, both experimental and theoretical, in the question of possible muon-number violation. This reflects the realization that in unified gauge theories, it is possible to account for the stringent experimental limits for processes such as $\mu \rightarrow e\gamma$, $\mu^- Z \rightarrow e^- Z$, $K_L \rightarrow e\mu$,...without having to require a fundamental law of muon number conservation, and moreover, that these processes could, in fact, occur with branching ratios which are not far from the present experimental upper limits.[5]

The existing pion factories were essential for obtaining the severe bounds on the branching ratios of strangeness-conserving muon number violating reactions and will continue to be indispensable for improving the obtained accuracies. What additional information one obtains from studies of the strangeness-changing processes? Would a facility capable of improving considerably the existing limits be of great significance? These are the issues I would like to try to explore here.[6]

Restricting attention to decay modes which do not involve neutrinos and/or photons, and which contain no more than three particles in the final state, the following muon-number violating, lepton-number conserving decay modes of the charged and neutral kaons are possible:[7]

$$K_L \to e^{\pm}\mu^{\mp}, \tag{1a}$$

$$K_S \to e^{\pm}\mu^{\mp}, \tag{1b}$$

$$K_L \to \pi^{o} e^{\pm}\mu^{\mp}, \tag{1c}$$

$$K_S \to \mu^{o} e^{\pm}\mu^{\mp}, \tag{1d}$$

$$K^{\pm} \to \pi^{\pm} e^{\pm}\mu^{\mp}. \tag{1e}$$

Needless to say, none of these have been seen so far. Experimental upper limits appear to be available only for (1a) and (1e).

Let us consider the decay $K_L \to \mu e$ ($K_L \to \mu^+ e^-$, for definiteness) in some detail. The general form of the amplitude is

$$M(K_L \to \mu^+ e^-) = A\ \bar{u}\gamma_5 v + B\ uv\ , \tag{2}$$

where A,B are complex numbers. Neglecting the electron mass, the decay rate is given by

$$\begin{aligned}\Gamma(K_L \to \mu^+ e^-) &= m_K(1 - m_\mu^2/m_K^2)^2(|A|^2 + |B|^2)/8\pi \\ &\simeq (1.8 \times 10^7)(|A|^2 + |B|^2)\ \text{eV},\end{aligned} \tag{3}$$

and the branching ratio relative to $K_L \to$ all is (using $\Gamma(K_L \to \text{all})_{exp} \simeq 1.27 \times 10^{-8}\,\text{eV}$)

$$\begin{aligned}B(K_L \to \mu^+ e^-) &\equiv \Gamma(K_L \to \mu^+ e^-)/\Gamma(K_L \to \text{all}) \\ &\simeq (1.4 \times 10^{15})(|A|^2 + |B|^2).\end{aligned} \tag{4}$$

To elucidate the meaning of the experimental bound,[7]

$$B(K_L \to \mu^+ e^-) < 2 \times 10^{-9}\ , \tag{5}$$

we shall have to consider the various ways in which muon number violation could take place. To remain as model-independent as possible, we shall first represent the muon number violating interaction involved by a phenomenological quark-lepton coupling of the form

$$\mathcal{L}_{eff} = -\frac{G}{\sqrt{2}}\ [(f_{VA}\ \bar{e}\gamma_\lambda\mu + f_{AA}\ \bar{e}\gamma_\lambda\gamma_5\mu)J_A^\lambda \tag{6}$$

$$+ (f_{SP}\ \bar{e}\mu + f_{PP}\ \bar{e}i\gamma_5\mu)J_P] + \text{H.c.},$$

where f_{VA}, f_{AA},...are parameters characterizing the strength of the corresponding terms relative to $2^{-1/2}$ G ($G \simeq 10^{-5}m_p^{-2}$) and

$$J_A^\lambda = \bar{s}\gamma_\lambda\gamma_5 d + \bar{d}\gamma_\lambda\gamma_5 s \tag{7}$$

$$J_P = \bar{s}i\gamma_5 d + \bar{d}i\gamma_5 s\ . \tag{8}$$

(6) is the most general nonderivative local effective Lagrangian that could contribute to $K_L \to \mu e$. As we shall see, it covers all the cases of interest.

The contributions of (6) to A,B are

$$A \simeq (4.2\times 10^{-7})f_{AA}\ a_A + (2\times 10^{-6})f_{PP}\ a_P\ , \tag{9}$$

$$B \simeq -(4.2\times 10^{-7})f_{VA}\ a_A - (2\times 10^{-6})if_{SP}\ a_P \tag{10}$$

where $a_{A,P}$ are defined by

$$\langle 0|J_\lambda^A|K_L(p)\rangle = p_\lambda\ m_K\ a_A/\ \sqrt{2m_K} \tag{11}$$

$$\langle 0|J^P|K_L(p)\rangle = -i\ m_K^2\ a_P/\ \sqrt{2m_K}\ . \tag{12}$$

The constant a_A can be estimated using SU(2) symmetry:

$$\langle 0|J_\lambda^A|K_L(P)\rangle = \sqrt{2}\ \langle 0|\bar{s}\gamma_\lambda\gamma_5 u|K^+\rangle \equiv f_K\ p_\lambda/\sqrt{2m_K} \tag{13}$$

while a_P can be related to a_A making use of the relation

$$\partial^\lambda J_\lambda^A = (m_s + m_d)J^P\ . \tag{14}$$

One finds

$$a_A \simeq 0.48 \tag{15}$$

$$a_P \simeq a_A m_K/(m_s + m_d) \simeq 1.5 \tag{16}$$

where we have used $f_K \simeq 1.23\ m_\pi$[8] and $m_s \simeq 150$ MeV, $m_d \simeq 7.5$ MeV for the quark masses.[9]

For simplicity, we shall consider two special cases for the Lagrangian (5):

a) $f_{VA} = f_{PP} = f_{SP} = 0;\ f_{AA} \neq 0$,

b) $f_{VA} = f_{AA} = f_{PP} = 0;\ f_{PP} \neq 0$.

The experimental bound (5), together with equations (9), (15), and (16) implies then

$$|f_{AA}| \lesssim 6\times 10^{-6} \tag{17}$$

$$|f_{PP}| \lesssim 4\times 10^{-7} \tag{18}$$

for cases (a) and (b), respectively.

Is there any further significant constraint on f_{AA}, f_{PP}? As we shall see, the answer is in general affirmative, since the interactions which lead to the effective Lagrangian (6) will, as a rule, also contribute to the $K_L - K_S$ mass difference $\Delta m \equiv m_L - m_S$, and Δm, being second-order weak in magnitude, is extremely sensitive to contributions from strangeness-changing neutral "current" interactions.

In the framework of unified gauge theories, $K_L \to \mu e$ (as well as other $\Delta S = 1$ muon-number violating processes) could occur via higher-order effects (μ,e coupled to intermixing leptons) or at the tree level, provided that there are neutral gauge bosons or neutral Higgs mesons coupled directly to both (μe) and (sd).[10]

Let us consider these possibilities individually:

1) Muon number violation via neutral gauge boson exchange. In a sequential $SU(2)_L \times U(1)$ gauge theory, flavour-changing neutral gauge boson-fermion couplings are absent.[11] However, such couplings may, in general, be present if $SU(2)_L \times U(1)$ was part of a larger flavour group. Quark-flavour and lepton-flavour changing transitions will also be present if the usual "vertical" gauge interactions are supplemented by "horizontal ones," which connect the generations, as may be necessary in order that the parameters of the mixing matrix, connecting the mass eigenstates and the gauge-group eigenstates, be calculable.[12]

Let us consider a fermion-gauge boson coupling of the form

$$\mathcal{L}_{fX} = g'\bar{e}\gamma_\lambda\gamma_5\mu\, X^\lambda + g''\, J^A_\lambda X^\lambda + \text{H.c.} \tag{19}$$

with J^A_λ given by eq. (7). The Lagrangian (19) leads to an effective semileptonic interaction (6), with $f_{SP} = f_{PP} = f_{VA} = 0$ and $f_{AA} = 8g'g''M_W^2/g^2M_X^2$, $(g^2/8M_W^2 = G/\sqrt{2})$, and consequently to $K_L \to \mu e$ with a branching ratio (cf.eqs. (3), (9), and (15))

$$B(K_L \to \mu e) = (3.7 \times 10^3)(g'g''/g^2)^2(m_W/m_X)^4 \ . \tag{20}$$

The Lagrangian (19) will also give a contribution

$$\Delta m_X = 2(g'')^2 \mathrm{Re}\, 2\langle \bar{K}^0 | (\bar{s}\gamma^\lambda\gamma_5 d)(\bar{s}\gamma^\lambda\gamma_5 d) | K^0\rangle / M_X^2 \tag{22}$$

to the $K_L - K_S$ mass difference Δm. There is no reliable method available at present to evaluate the matrix element (22). An estimate, which should be adequate for our purposes, can be obtained using the "vacuum insertion method."[13] We find

$$\Delta m_X = \frac{8}{3} f_K^2 m_K (g''/M_X)^2 \tag{23}$$

$$\simeq (2.5 \times 10^3)(g''/g)^2(M_W/M_X)^2 \ \text{eV} \ .$$

For Δm_X not to exceed the experimental value $\Delta m_{exp} \simeq (3.5 \times 10^{-6})$ eV, we must have

$$M_X \gtrsim (2.7 \times 10^4)\, \frac{g''}{g}\, M_W \ . \tag{24}$$

As a consequence, for a given g' and g'', $B(K_L \to \mu e)$ cannot be arbitrarily large, but must obey[14]

$$B(K_L \to \mu e) \lesssim (7 \times 10^{-15})(g'/g'')^2 \ . \tag{25}$$

It follows that $g'/g'' \lesssim 500$, $B(K_L \to \mu e)$ reaching its experimental upper limit (5) for $g' \approx 500\ g''$.[15] If we assume (in the spirit of unified gauge theories) that $g' \approx g'' \approx g$, then

$$M_X \gtrsim (2.7 \times 10^4) M_W \ , \tag{26}$$

$$|f_{AA}| < 10^{-8} \ , \tag{27}$$

and

$$B(K_L \to \mu e) \lesssim 7 \times 10^{-15} \ . \tag{28}$$

2) Muon number violation via neutral Higgs exchange. If the Higgs sector of the $SU(2)_L \times U(1)$ model is extended to include at least two Higgs doublets, muon number may be violated by the Higgs-lepton couplings.[16] The neutral Higgs mesons which mediate $\mu \leftrightarrow e$ transitions could, in general, be coupled also to $\Delta S = 1$ quark densities, leading to processes such as $K_L \to \mu e$. Muon number may be violated also by Higgs mesons associated with group structures beyond $SU(2)_L \times U(1)$.

Let us consider a Higgs-fermion interaction of the form

$$\mathcal{L}_{fh} = g_h'(\bar{s}i\gamma_5 d + \bar{d}i\gamma_5 s)\phi_h + g_h''\ \bar{e}i\gamma_5\mu\phi_h \ . \tag{29}$$

The contribution of (29) to $B(K_L \to \mu e)$ and Δm is given by

$$\begin{aligned} B(K_L \to \mu e) &\approx (1.3 \times 10^4)|f_{PP}|^2 \\ &\approx (1.3 \times 10^4)(\sqrt{2}\ g_h' g_h''/G m_h^2)^2 \end{aligned} \tag{30}$$

and

$$\Delta m_h = \frac{8}{3}\ (g_h''/m_h)^2 m_K f_K^2 [m_K/(m_s + m_d)]^2 . \tag{31}$$

Δm_{exp} and eq. (31) imply

$$m_h > 10^7 g_h''\ \mathrm{GeV} \tag{32}$$

so that for a given g_h' and g_h'', we must have

$$B(K_L \to \mu e) \lesssim (1.6 \times 10^{-14})(g_h'/g_h'')^2 \ . \tag{33}$$

Consequently, the experimental limit (5) requires $g_h'/g_h'' \lesssim 350$. As an example, suppose that $g_h' = 2^{1/4} m_\mu \sqrt{G}$ and $g_h'' = 2^{1/4} m_s \sqrt{G}$. It follows that

$$m_h > (6 \times 10^3)\ \mathrm{GeV} \tag{34}$$

$$B(K_L \to \mu e) < 8 \times 10^{-15} \ . \tag{35}$$

The choice $g_h' = 2^{1/4} m_\mu \sqrt{G}$, $g_h'' = 2^{1/4} m_d \sqrt{G}$ would, instead, lead to

$$m_h > 320\ \mathrm{GeV} \tag{36}$$

$$B(K_L \to \mu e) < 3 \times 10^{-12} \ . \tag{37}$$

Larger values of $B(K_L \to \mu e)$ corresponding to other ratios for the Higgs couplings cannot be, of course, ruled out.

3) Muon number violation via intermixing leptons. A prominent example here is the standard sequential six quark-six lepton $SU(2)_L \times U(1)$ model, which has so far been remarkably successful in accounting for a wide variety of weak interaction data. Even in the absence of the mechanisms described earlier, muon number will not be in general conserved as long as the neutrinos are not all exactly massless (or degenerate). Neglecting muon number violation due to ν_e, ν_μ mixing, all muon number violating effects will be proportional to the parameter $\beta\gamma$ where β and γ measure the amount of the ν_τ mass eigenstate in the gauge-group eigenstates ν_e' and ν_μ', respectively.[17,18] The $K_L \to \mu e$ rate in this model has been calculated in ref. 17. With the experimental limits $(\beta\gamma)^2 < 2 \times 10^{-3}$ [17,18] and $m_{\nu_\tau} < 250$ MeV[19] one obtains

$$B(K_L \to \mu e) < 5 \times 10^{-16} \ . \tag{38}$$

We are now ready to state our conclusions regarding $K_L \to \mu e$:
a) Above the level of about 10^{-14} for the branching ratio, the decay $K_L \to \mu e$ is not expected to be sensitive to a flavour changing neutral gauge boson coupled to (μe) and (sd) with comparable strength. In contrast, a neutral gauge boson coupled to (μe) and to a strangeness-conserving quark density only, the rate for $\mu^- Z \to e^- Z$ could be as large as the corresponding experimental limit.[20] Consequently, in the range $10^{-14} < B(K_L \to \mu e) < 2 \times 10^{-9}$ the decay $K_L \to \mu e$ probes the presence of other possible sources of muon number violation. b) A branching ratio considerably larger than $\sim 10^{-12}$ would suggest any of the following possibilities or combinations thereof: the existence of an additional generation of leptons and quarks[21]; the presence of a neutral Higgs boson[22] coupled to (μe) more strongly or to (sd) more weakly than we have assumed above; the existence of a strangeness and muon number changing neutral guage boson with a considerably stronger coupling to (μe) than to (sd).

Concerning $K_S \to \mu e$, no experimental information seems to be available yet. This decay is sensitive to the antisymmetric combination $\bar{s}\Gamma_i d - \bar{d}\Gamma_i s$ $(i = A,P)$. In general, (μe) may be coupled to a linear combination $a(\bar{s}\Gamma d) + b(\bar{d}\Gamma s)$ with $a \neq \pm 1$, allowing for both $K_L \to \mu e$ and $K_S \to \mu e$ to occur.

We shall turn now to consider briefly $K^+ \to \pi^+ \mu e$. This decay would be sensitive to an effective interaction analogous to (6) but involving vector, scalar, or tensor[23] quark densities. For a vector coupling the branching ratio is

$$B(K^+ \to \pi^+ \mu e) \equiv \Gamma(K^+ \to \pi^+ \mu e)/\Gamma(K^+ \to \text{all}) \simeq (6.4 \times 10^{-1})|f_{VV}|^2 \ . \tag{39}$$

Assuming that the contribution to Δm of a neutral gauge boson X_V^λ coupled to $J^V = \bar{s}\gamma_\lambda d + \bar{d}\gamma_\lambda s$ is comparable to Δm_X (eq. 23),[8] one obtains

$$B(K^+ \to \pi^+ \mu e) < (2 \times 10^{-17})(g_V'/g_V'')^2 , \tag{40}$$

to be compared with the experimental limit[7]

$$B(K^+ \to \pi^+ \mu e)_{exp} < 7 \times 10^{-9} \ . \tag{41}$$

For a scalar coupling,

$$B(K^+ \to \pi^+\mu e) = 8|f_{SS}|^2 \ , \tag{42}$$

and Δm_{exp} implies (assuming $\Delta m_S \simeq \Delta m_P$),

$$B(K^+ \to \pi^+\mu e) < 10^{-17}(g_S'/g_S'')^2 \quad . \tag{43}$$

For $g_S'/g_S'' = m_\mu/m_s$ and $g_S'/g_S'' = m_\mu/m_d$, (42) leads to

$$B(K^+ \to \pi^+\mu e) < 6 \times 10^{-18}$$

and

$$B(K^+ \to \pi^+\mu e) < 2 \times 10^{-15} \tag{45}$$

respectively.

In the standard $SU(2)_L \times U(1)$ model with three generations, one would have $B(K^+ \to \pi^+\mu e) < 6 \times 10^{-18}$.[17]

As seen from eqs. (39) - (46), compared to the results for $K_L \to \mu e$, $K^+ \to \pi^+\mu e$ is much less sensitive to a vector or scalar coupling than $K_L \to \mu e$ is to an axial vector or a pseudoscalar one.

No experimental limits are available for $K_{L,S} \to \pi^0\mu e$. Both will occur (as well as $K^+ \to \pi^+\mu e$, to which they can be related), if (μe) is coupled to a general combination $a\bar{s}\Gamma_i d + b\bar{d}\Gamma_i s$ $(i = V,S,T)$.

CP, T, and CPT

Although the discovery of CP-violation dates back to 1964, its origin remains an unresolved question. The experimental developments since 1964 can be summarized as follows:[24] 1) No CP-violation (or T-violation) was found outside the neutral kaon system; 2) More accurate experimental information became available on the parameters which describe the observed CP-violation; 3) Sharper limits were set on CP or T-violating amplitudes in various processes. The data are consistent with a theory in which CP-violation arises solely through the mixing of K_1^0 and K_2^0, such as a superweak theory.[25] On the theoretical side, the success of unified gauge theories led to investigations of the possible ways in which the observed CP-violation could arise in such a framework.[26] The basic mechanisms are CP-violation in the gauge boson-quark couplings and in the interactions of Higgs mesons. Below we shall discuss briefly two models which are of immediate experimental interest.

1) CP-violation in the gauge boson interactions. The prominent example is again the sequential $SU(2) \times U(1)$ model with three generations (the minimal number that could accommodate CP-violation in the fermion-gauge boson couplings).[27] CP-violation could reside also in the couplings of gauge bosons belonging to a larger flavour group.[28]

In the six quark-six lepton model, the mixing matrix, which relates the quark mass-eigenstates d,s,b to the gauge group eigenstates d',s',b', contains three mixing angles θ_1, θ_2, θ_3 and a CP-violating phase, δ:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_1 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2c_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$.

The parameters ε and ε' involved in the CP-violating observables[29]

$$\eta_{+-} = \varepsilon + \varepsilon' \tag{46}$$

$$\eta_{00} \simeq \varepsilon - 2\varepsilon' \tag{47}$$

are predicted in this model to be[30]

$$\sqrt{2}|\varepsilon| = (1-D)(\mathrm{Im}A_0/\mathrm{Re}A_0 + K'F) \ , \tag{48}$$

$$\sqrt{2}|\varepsilon'| = (1/20)\ \mathrm{Im}A_0/\mathrm{Re}A_0 \tag{49}$$

and

$$\mathrm{Im}A_0/\mathrm{Re}A_0 = fK'\tilde{F} \ . \tag{50}$$

In eqs. (48) - (50), A_0 is the $K \to 2\pi(I = 0)$ amplitude, $K' = s_2c_2t_3 \sin\delta/c_1$, ($t_i \equiv \tan\theta_i$) and $f\mathrm{Re}A_0$ is the fraction of $\mathrm{Re}A_0$ due to penguin diagrams. F and $\tilde{F}$ are functions of m_c/m_t and $K = s_2^2 + s_2c_2t_3\cos\delta/c_1$. $\tilde{F}$, in addition, depends on the cutoff mass μ in the evaluation of the penguin diagrams. $(1-D)\Delta m \equiv \Delta m_{box}$ is the usual box-diagram contribution to $\Delta m \equiv m_L - m_S$ and $D\Delta m$ is the part of Δm arising from contributions of low-mass intermediate states ($\pi^0, \eta, 2\pi, \ldots$), which cannot be reliably calculated.

For $f \approx 0$, apart from some new effects in rare kaon decays and in the decays of charmed particles, the model reproduces closely the results of the superweak theory.[31] An important result, arrived at in ref. 32 is that for an appreciable value of f (which may be the clue for the understanding of the $\Delta I = 1/2$ rule), the ratio

$$|\varepsilon'/\varepsilon| = f\tilde{F}/(1-D)(f\tilde{F}+F) \tag{51}$$

could be as large as the present experimental limit $|\varepsilon'/\varepsilon|_{exp} \lesssim 1/50$.[24]

For a given value of other parameters, (51) turns out to be a decreasing function of K. The maximum allowed value of K, dictated by D, corresponds to the minimum of $|\varepsilon'/\varepsilon|$. Assuming $|D| < 2$ (suggested by estimates of $D\Delta m$) and choosing $f \sim 1/2$, $m_c^2/\mu^2 = 2.25$, $m_t/m_c = 10$, the limits $|\varepsilon'/\varepsilon| \gtrsim 7.7 \times 10^{-3}$ or $|\varepsilon'/\varepsilon| \gtrsim 9 \times 10^{-3}$ are obtained,[30] depending on whether the box diagram was calculated by vacuum insertion, or in the bag model.[8] Smaller values of $|\varepsilon'/\varepsilon|$ (which would correspond to larger values of one or more of the quantities m_t/m_c, m_c/μ, and D, or to a smaller value of f[30,32]) cannot be, of course, ruled out.

In addition to effects in $K \to 2\pi$, deviations from the superweak prediction may occur also in some other nonleptonic decays, for example, in $K \to 3\pi$. Predictions of the possible magnitude of such effects are yet to be made.[33] In semileptonic decays no CP-violation is present in lowest order. The electric dipole moment of the neutron D_n is predicted to be of the order of 10^{-29} to 10^{-30}, (to be compared with the present experimental limit $|D_n|_{exp} < 1.6 \times 10^{-24}$ (90% C.L.))[34] which presumably would not be attainable even with the next generation of experiments.

2) CP-Violation through Higgs boson exchange. If the Higgs sector of the $SU(2)_L \times U(1)$ model is extended to include at least three doublets (at least two, if the neutral Higgs exchange is allowed to change flavour), the quartic self interaction of the Higgs bosons may, in general, violate CP.[35] CP will not be then conserved by the scalar propagators and as a result, the exchange of a higgs boson will induce an effective CP-violating Fermi interaction with strength of order Gmm'/m_H^2.[35]

CP-violating effects in this model have been estimated in ref. 37 with the results $|\varepsilon'/\varepsilon| \simeq 0.02(!)$ and $D_n = -2.8 \times 10^{-25}$. It should be noted that by increasing the "transition" Higgs mass, D_n could be made correspondingly smaller, but the model could not then account for the observed CP-violation. The latter, on the other hand, may be due to other causes. In order that $|\varepsilon'/\varepsilon|$ could have a smaller magnitude, further Higgs doublets seem necessary. CP-violating effects are expected also in semileptonic reactions, notably a muon polarization P_μ normal to the decay plane in $K_L^0 \to \pi^-\mu^+\nu$ and $K^+ \to \pi^0\mu^+\nu$, corresponding to $\mathrm{Im}\xi$ (the ratio of T-violating and T-invariant form factors) of order 10^{-3}, to be compared with the present experimental limit $\mathrm{Im}\xi = 0.012 \pm 0.026$.[38] Again, smaller values of $\mathrm{Im}\xi$ cannot be ruled out. In particular, CP-violating effects in $K_L \to 2\pi$ and in $K_{\mu 3}$ may not be related, since in the leptonic couplings a different Higgs meson might be involved. To search for smaller P_μ, $K^+ \to \pi^0\mu^+\nu$ is the better suited, since here the final-state interactions are relatively negligible.

In addition to CP-violating quantities such as η_{+-} and η_{00}, we have also encountered in models described above observables which violate time-reversal invariance. The simultaneous appearance of CP-violation and T-violation is of course a consequence of the CPT theorem, which is satisfied in a local relativistic quantum field theory, such as the models we have considered. Although the success of gauge theories has but strengthened our belief in local quantum field theory as the correct framework to describe the fundamental interactions, it is important to remain open minded and to be aware of the extent to which consequences such as the CPT theorem have been experimentally tested.

As emphasized in ref. 39, the existing bounds on the strength of possible CPT-violating interactions depend on their symmetry properties. The best available limit refers to the case when the CPT-violating interaction is also CP-violating, but conserves parity and strangeness. It is deduced from the limit on the mass difference between K^0 and $\bar{K}^0$, $S_K \equiv [m(K^0) - m(\bar{K}^0)]/(m_L - m_S)$.[39] Let us write for the parameter ε (in eq. 46) $\varepsilon = \tilde{\varepsilon} + \tilde{\delta}$, where $\tilde{\varepsilon}$ and $\tilde{\delta}$ represent the CPT invariant (T-violating) and CPT-violating (T-invariant) parts, respectively. An evaluation[40,24] of the Bell-Steinberger unitarity relation, using the present experimental information yields[24] $\mathrm{Re}\tilde{\varepsilon} = (1.61 \pm 0.25) \times 10^{-3}$, $\mathrm{Im}\tilde{\varepsilon} = (1.40 \pm 0.25) \times 10^{-3}$, $\mathrm{Re}\tilde{\delta} = (-0.03 \pm 0.27) \times 10^{-3}$, and $\mathrm{Im}\tilde{\delta} = (-0.23 \pm 0.27) \times 10^{-3}$ implying that the observed CP-violation is predominantly due to a T-violating (CPT-invariant) interaction! The constant $\tilde{\delta}$ is related to δ_K as
(CPT-invariant) interaction! The constant $\tilde{\delta}$ is related to δ_K as
$\sqrt{2}\tilde{\delta} = 1/2\delta_K + \beta_0$[39] where $\beta_0 \simeq [\gamma(K^0) - \gamma(\bar{K}^0)]/[\gamma(K^0) + \gamma(\bar{K}^0)]$ ($\gamma(K^0)$ and $\gamma(\bar{K}^0)$ are the decay width of K^0 and $\bar{K}^0$).[39] Assuming that $\beta_0 \simeq \Delta_K \equiv [\gamma(K^+) - \gamma(K^-)]/[\gamma(K^+) + \gamma(K^-)]$, one infers from the experimental bound

$\Delta_K \leq 10^{-3}$ [41] that $|\delta_K| \leq 4 \times 10^{-3}$. Denoting the strength of the hypothetical CPT-violating interaction by μG (G = Fermi constant), its contribution to δ_K is of order $|\delta_K| \simeq \mu G/G^2 = \mu/G$ and consequently

$$\mu \lesssim (4 \times 10^{-3})G \tag{52}$$

implying that the strength of the CPT-violating interaction must be of the order of the superweak interaction or smaller. The limit could be improved by better information on CP-violating K^o decay modes and on Δ_K (provided that the assumption $\beta_0 \simeq \Delta_K$ holds[39]). If the CPT-violating, CP-violating interaction violates parity or strangeness (or both), the information on its strength is weaker: $|\delta_K| \simeq \mu G^2/G^2 \simeq \mu < 4 \times 10^{-3}$. A limit of $|\delta_K| < 10^{-3}$ is obtained from Δ_K (or from $\Delta_\pi < 10^{-3}$,[42] for the S = 0 case).[39] Thus in this case CPT-violation at the level of a miliweak interaction is still tolerable! The limit could be improved by a more accurate experimental result on Δ_K. For CP-conserving, CPT-violating interactions, the limit from δ_K, Δ_K, or Δ_π depends on the strength of the CP-violating interaction. In most cases the best limits are obtained instead from direct tests of time-reversal violation.[39]

CONCLUSIONS

The main subjects of our discussion were muon number violating processes with change of strangeness, and some new developments in the field of CP-violation. Although unified gauge theories provide a number of possible mechanisms for the breakdown of muon number conservation, muon number violation is not obligatory, and its discovery would be an event of great importance. On the basis of our discussion it appears that the chances for muon number violation to be found, with a branching ratio between about 10^{-14} and the existing experimental limits, are better for the $\Delta S = 0$ reaction $\mu^- Z \to e^- Z$ than for the $\Delta S = 1$ processes we considered. Once muon number violation was found, however, studies of the $\Delta S = 1$ processes would provide new information on the underlying mechanisms. Independently on whether muon number violation is seen or not, improved experimental limits on $\Delta S = 1$ muon number violating processes would give important constraints, independent on those imposed by the leptonic and the $\Delta S = 0$ semileptonic reactions, on the possible muon number violating interactions.

Unlike muon number violation, the breakdown of CP-symmetry is an established fact. The aim of the experiments is to identify the interaction responsible for it. The prospect that one may be able to distinguish some of the possible sources of CP-violation from a CP-violating superweak interaction is a most exciting one, and to improve the accuracy of the relevant experiments is of the greatest importance. Some progress can already be made by the existing machines,[43] but for the final answer we may have to wait for new, high intensity beams of kaons.

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15. A conceivable situation (which does not seem to be however very attractive) that could lead to $K_L \to \mu e$ with vastly different magnitudes for g' and g" would be the existence of separate horizontal gauge groups for leptons and quarks, with gauge bosons X^l and X^h which mix with each other.
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vector coupling to (sd). $K_L \rightarrow \mu e$ would occur with a branching ratio $B(K_L \rightarrow \mu e) = (3.7 \times 10^{[illegible]})(M_W/M_X)^4$. Thus, both processes are about equally sensitive to M_X. But M_X now must obey eq. 26, implying $R^{coh}_{\mu e} \lesssim 9 \times 10^{-15}$ and $B(K_L \rightarrow \mu e) \lesssim 7 \times 10^{-15}$ (we have assumed that all the gauge couplings involved are equal to g).

21. Note that if muon number is violated via intermixing leptons, $\Gamma(K_L \rightarrow \mu e)$ depends, unlike $\Gamma(\mu^- Z \rightarrow e^- Z)$, also on the quark masses.
22. Assuming $g'_h = 2^{1/4}\sqrt{G}\, m_\mu$, and a $\Delta S = 0$ quark-Higgs coupling of the form $2^{1/4}\sqrt{G}\,(m_u \bar{u}u + m_d \bar{d}d)\phi_H$, the coherent $\mu^- \rightarrow e^-$ capture rate for sulfur is (using the formula of O. Shanker, ref. 20) $R^{coh}_{\mu e} = (2.8 \times 10^2) \times [m_\mu(m_u + m_d)m_H^2]^2$. The experimental limit on $R_{\mu e}$ implies $m_H > 50$ GeV. If the same Higgs meson couples also to (sd) with a coupling $g''_h \approx 2^{1/4}\sqrt{G}\, m_d$, then $R_{\mu e} \rightarrow 4 \times 10^{-14}$, while $B(K_L \rightarrow \mu e) < 3 \times 10^{-12}$.
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